Fertility Subsidies Can Have Ambiguous Effects on Birth Timing*

Jesse Naidoo†

Abstract

Pronatalist subsidies often vary with birth order (parity). I study the effect of such subsidies on birth timing in a life-cycle model of fertility choice. In the model, births permanently reduce the rate of human capital accumulation. While subsidies to marginal births always accelerate the time to next birth, subsidies to higher-order births can extend those times for women at low parities. The result is not driven by income effects, quantity-quality substitution, biological constraints, or uncertainty. Instead, it is that slower anticipated earnings growth in the future raises the marginal value of human capital in the present.

KEYWORDS: fertility, life-cycle, birth spacing, human capital.

JEL CODES: J13, D15.

***Current version**: June 06, 2023. **First version:** May 2022.

†Department of Economics, University of Pretoria, Hatfield 0002, South Africa. **email:** jesse.naidoo@up.ac.za. I thank Alex Zimper, Steve Koch, Taryn Dinkelman, and Daniel Kaliski for helpful comments. **Declarations of interest:** none. **This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.**

1 Introduction

Aggregate fertility rates have been declining across the world for several decades, with several countries in East Asia and Europe falling well below replacement levels (Kohler, Billari, and Ortega (2002)). This decline has been coupled with a shift in the timing of childbearing to later ages (Beaujouan (2020)).

In response, many governments have introduced pronatalist subsidies. These payments to parents often vary with birth order. For example, Cohen, Dehejia, and Romanov (2013) documents how Israel's child allowance is the same for first and second children, but payments increase sharply with parity for third and higher-order births. Similar features are present in Russia's "maternity capital" programme (Malkova (2018)) and Quebec's "Allowance for Newborn Children" (Parent and Wang (2007)).

In this note, I set out a simple life-cycle model of fertility timing in which each subsequent birth permanently reduces a woman's rate of human capital accumulation. I show that introducing subsidies to specific births can have ambiguous effects on birth timing across the distribution of parity. While subsidies to marginal births always accelerate the time to next birth, subsidies to higher-order births can extend those times for women at low parities. This is because those at low parities foresee slower human capital growth in their future, and respond by accumulating more in the present.

Previous attempts to model life-cycle fertility are not well suited to this question. Both Happel, Hill, and Low (1984) and Blackburn, Bloom, and Neumark (1993) build models of fertility timing, but for first births only. Cigno and Ermisch (1989) also provides a model of life-cycle fertility choice. However, that paper models the choices of a representative agent who chooses aggregate birth *rates*, but who does not keep track of the distribution of birth histories within a cohort.

Many empirical papers estimate statistical models of fertility timing, such as Barmby and Cigno (1990) or Heckman and Walker (1990). Others propose economic models of life-cycle fertility that involve more complex mechanisms than mine. For example, Adda, Dustmann, and Stevens (2017) constructs a dynamic model of fertility featuring income effects in labour supply. The presence of income effects allows both credit constraints and wealth shocks to matter for fertility timing. There are no income effects in this model; nor is there a "quality" margin or any biological constraints on birth timing. While those mechanisms may indeed be important in explaining some aspects of fertility timing, they are not necessary to establish the ambiguities that are my interest here.

2 A Life-Cycle Model of Fertility Choice

A woman who lives forever may have up to K children, where $K \geq 2$ is an integer. She cares about consumption c_t and family size k_t at each point in time. She discounts the future at rate ρ and receives flow utility $u(c_t, k_t)$, where $u(c_t, 0) = c_t$ and for $k \geq 1$,

$$u(c_t, k_t) = c_t + \sum_{j=1}^{k_t} \theta_j.$$

Here, $\theta_j \geq 0$ represents the flow of benefits from the j-th child (the "joy of parenthood"), net of any out-of-pocket expenditures.

With human capital of $h_t \in [0, 1]$, her instantaneous earnings are wh_t, where $w > 0$ is a given skill price. Between births, human capital accumulates at the rate $\dot{h}_t = \gamma_k(1 - h_t)$, for some $\gamma_k > 0$. I assume $\gamma_k > \gamma_{k+1}$ for all k, which is necessary for the second-order conditions to hold.

The state for a woman's decision problem is (k, h). If she commits to never having any further children - which will in general not be optimal - she receives the value

$$\overline{V}_k(h) \quad = \quad \rho^{-1}\sum_{j=1}^{k}\theta_j + w\left[\rho^{-1} - (1-h)(\rho+\gamma_k)^{-1}\right]. \tag{1}$$

For $k = K$, no further births are feasible, so her value function is simply $V_K(h) = \overline{V}_K(h)$. For $1 \le k \le K-1$, her value function is

$$V_k(h) = \max_{h_k,\, S_k} \int_0^{S_k} e^{-\rho t}\left[wh_t + \sum_{j=1}^{k}\theta_j\right] dt + e^{-\rho S_k} V_{k+1}(h_k)$$

subject to the constraint $h_k \le 1-(1-h)e^{-\gamma_k S_k}$. (For $k = 0$, the obvious adjustment applies.) Her age at the k-th birth is the sum of the prior interbirth intervals: $T_k = S_0 + S_1 \ldots + S_{k-1}$.

Since h is increasing and childbirth is irreversible, the only feasible policy is of the following form: for some threshold h_k^*, a woman of parity k will optimally choose to have her $(k+1)$-st child if $h \ge h_k^*$, and wait if $h < h_k^*$.

The Hamilton-Jacobi-Bellman equation for a woman of parity k is

$$\rho V_k(h) \quad = \quad wh + \sum_{j=1}^{k}\theta_j + \gamma_k(1-h)V_k'(h) \tag{2}$$

which holds for $h < h_k^*$. For $h \ge h_k^*$, we have $V_k(h) = V_{k+1}(h)$. The value matching and smooth pasting conditions state that V_k and V_k', respectively, must be left-continuous at h_k^*:

$$\lim_{h\to h_k^*+} V_k(h) \quad = \quad V_{k+1}(h_k^*) \tag{3}$$

$$\lim_{h\to h_k^*+} V_k'(h) \quad = \quad V_{k+1}'(h_k^*). \tag{4}$$

When spacing is nonzero at all parities, all value functions have to be convex.

LEMMA 1. *If $h_k^* < h_{k+1}^*$ for all k, then $V_k(h)$ is weakly convex in h for all k.*

The proof follows by induction on parity; details are given in the appendix. Some intuition for this result comes from the convergence properties of the law of motion for h: early in one's career (when human capital is low), earnings growth is fast, but slows down later. Thus the marginal value of human capital is higher later in the career, because further growth is harder to obtain.

Manipulating (3) and (4) leads to the following.

LEMMA 2. *If $h_k^* < h_{k+1}^*$, then h_k^* is the unique solution to the equation*

$$\theta_{k+1} \quad = \quad (\gamma_k - \gamma_{k+1})(1-h_k^*)V_{k+1}'(h_k^*). \tag{5}$$

Notice that θ_{k+1} is the marginal cost of delaying the $(k+1)$-st birth by a short length of time. The marginal benefit is $(\gamma_k - \gamma_{k+1})(1-h)V_{k+1}'(h)$, the effect of a birth on the flow of capital gains from waiting. Uniqueness follows from the convexity of the value functions.

3 Effects of Fertility Subsidies

Because preferences are linear in consumption, a cash payment conditional on birth j is equivalent to an increase in θ_j. So understanding the effects of fertility subsidies amounts to finding the comparative statics $\partial h_k^*/\partial \theta_j$. From now on we ignore corner solutions, which is valid under some restrictions on preferences (described in full in the appendix).

Using the HJB equation (2) and the fact that optimal thresholds satisfy (5), we can obtain the comparative statics for h_k^* by implicitly differentiating the condition

$$\rho V_{k+1}(h_k^*) - \left(w h_k^* + \sum_{j=1}^{k+1} \theta_j \right) \equiv \frac{\theta_{k+1}\gamma_{k+1}}{\gamma_k - \gamma_{k+1}}.$$

Doing so and applying the envelope theorem delivers the following result.

Proposition 1. *A marginal increase in the payoff to a j-th birth, θ_j, affects the human capital threshold at parity k as follows:*

$$\frac{\partial h_k^*}{\partial \theta_j} = \begin{cases} e^{-\rho(T_j^* - T_{k+1}^*)}[w - \rho V'_{k+1}(h_k^*)]^{-1} & \text{if } j > k+1 \\ -\gamma_{k+1}(\gamma_k - \gamma_{k+1})^{-1}[w - \rho V'_{k+1}(h_k^*)]^{-1} & \text{if } j = k+1 \\ 0 & \text{if } j \le k \end{cases}$$

(Notice that Lemma 1 implies $V'_{k+1}(h_k^*) \le \overline{V}'_K(h_k^*) = w(\rho + \gamma_K)^{-1} < w\rho^{-1}$.) Given the absence of any income effects, subsidising inframarginal births does not affect current decisions: bygones are bygones. Subsidies to the marginal birth ($j = k+1$) lower the threshold h_k^*. However, subsidies to higher-order births *raise* h_k^*, slowing down the time to $(k+1)$-st birth.

To understand why, note the envelope theorem implies $V'_{k+1}(h)$ depends only on θ_j for $j > k+1$. In particular, for $j > k+1$, the cross-partial derivative is positive:

$$\frac{\partial^2 V_{k+1}}{\partial h \partial \theta_j}(h) = \frac{\partial}{\partial h} e^{-\rho S_{k+1}^*(h)} \cdot e^{-\rho(T_j^* - T_{k+2}^*)} = e^{-\rho(T_j^* - T_{k+2}^*)} \cdot \frac{\partial}{\partial h}\left(\frac{1 - h_{k+1}^*}{1-h}\right)^{\rho/\gamma_{k+1}} > 0.$$

Thus subsidies to higher-order births make human capital *more* valuable at the margin today, and this raises the marginal benefit of delaying the current birth. The intuition here is that a forward-looking woman recognises that in the future, she will have stronger incentives to accelerate subsequent births. And if human capital growth in the future will be slower, it is more important to accumulate it in the present.

4 Explicit Solution for a Special Case

When a woman can have at most two children, and $\rho = \gamma_1$, the optimal thresholds h_0^* and h_1^* can be expressed analytically. We will look for an interior solution.

If $h_1^* < 1$, then also $\theta_2 = (\gamma_1 - \gamma_2)(1 - h_1^*)w(\rho + \gamma_2)^{-1}$. For notational convenience, let $z = 1 - h$. Then we have

$$z_1^* \quad = \quad \left(\frac{\theta_2}{w}\right)\left(\frac{\rho + \gamma_2}{\gamma_1 - \gamma_2}\right). \tag{6}$$

As expected, h_1^* is decreasing in θ_2/w. And we only have $h_1^* > 0$ if $\theta_2/w < (\gamma_1 - \gamma_2)/(\rho + \gamma_2)$: if preferences for a second child are strong enough, it will never be optimal to wait for a second birth.

The value function $V_1(h)$ is

$$V_1(h) = \int_0^{S_1^*(h)} e^{-\rho t}[w(1-(1-h)e^{-\gamma_1 t}) + \theta_1]dt + e^{-\rho S_1^*(h)} V_2(h_1^*)$$

After some algebra, this simplifies to

$$V_1(h) = \begin{cases} \rho^{-1}(\theta_1 + w) - w(\rho+\gamma_1)^{-1}(1-h) + \rho^{-1}\theta_2\left(\frac{1-h_1^*}{1-h}\right) - \theta_2(\rho+\gamma_1)^{-1}\left(\frac{1-h_1^*}{1-h}\right) & \text{if } h < h_1^* \\ V_2(h) & \text{if } h \geq h_1^*. \end{cases}$$

We need the assumption that $\rho = \gamma_1$ to obtain the above expression. Under it, the equation determining z_0^* is quadratic: if $h_0^* < h_1^*$,

$$\theta_1 = (\gamma_0 - \gamma_1)\{w(\rho+\gamma_1)^{-1}z + \theta_2(\rho+\gamma_1)^{-1}z_1^* \cdot z^{-1}\}. \tag{7}$$

A solution z_0^* to (7) lies in $(z_1^*, 1)$ if and only if

$$\left(\frac{\theta_2}{w}\right)\left(\frac{\gamma_0-\gamma_1}{\gamma_1-\gamma_2}\right) < \theta_1/w < \left(\frac{\gamma_0-\gamma_1}{\rho+\gamma_1}\right)\left[1 + \left(\frac{\theta_2}{w}\right)^2\left(\frac{\rho+\gamma_2}{\gamma_1-\gamma_2}\right)\right].$$

Further, if $z_0^* \in (z_1^*, 1)$, the right-hand side of (7) is increasing in z at z_0^*. Thus, the second-order conditions will hold. The optimal thresholds are then

$$\begin{aligned} z_0^* &= \frac{1}{2}\left\{\frac{\theta_1(\rho+\gamma_1)}{w(\gamma_0-\gamma_1)} + \sqrt{\left(\frac{\theta_1}{w}\right)^2\left(\frac{\rho+\gamma_1}{\gamma_0-\gamma_1}\right)^2 - 4\left(\frac{\theta_2}{w}\right)^2\left(\frac{\rho+\gamma_2}{\gamma_1-\gamma_2}\right)}\right\} \\ z_1^* &= \left(\frac{\theta_2}{w}\right)\left(\frac{\rho+\gamma_2}{\gamma_1-\gamma_2}\right). \end{aligned}$$

Notice how z_0^* is decreasing in θ_2, which means that h_0^* is *increasing* in θ_2. Finally, $V_0(h)$ can be expressed as

$$V_0(h) = \int_0^{S_0^*(h)} e^{-\rho t} w(1-(1-h)e^{-\gamma_0 t})dt + e^{-\rho S_0^*(h)} V_1(h_0^*).$$

5 Concluding Remarks

The solution of the model induces a mapping from a woman's initial human capital and her vector of preference parameters, θ, to her path of births and earnings. Proposition 1 implies that this mapping is locally invertible. Thus, if a set of women facing common labour market conditions differ in their preferences according to some nondegenerate distribution for θ, the implied distribution of their birth timings will also be nondegenerate.

An econometrician with who does not observe preferences will therefore not be able to predict birth timings perfectly, despite the fact that at the individual level, life-cycle choices play out deterministically.

If one has data in which in parity-specific subsidies vary exogenously, the predictions of Proposition 1 could be tested using standard techniques, even without additional shocks. (Imagine, for example, an experiment that exposed a random subset of a given cohort to such a subsidy.)

Assuming that h follows a diffusion process would be a natural extension of the model. Whether the results of Proposition 1 are preserved will depend on how the marginal value of human capital varies with the characteristics of the terminal payoff function. In some cases, e.g. if the process is mean-reverting, the value function for this optimal stopping problem can have both concave and convex regions, so ambiguity is not out of the question, but this remains to be seen.

A Appendix

A.1 Convexity of Value Functions

Proof of Lemma 1. The problem is

$$V_k(h) = \max_{h_k,\, S_k} \int_0^{S_k} e^{-\rho t} \left[w h_t + \sum_{j=1}^{k} \theta_j \right] dt + e^{-\rho S_k} V_{k+1}(h_k)$$

subject to $h_k = 1 - (1-h)e^{-\gamma_k S_k}$, where h is the given initial level of human capital. Using λ for the multiplier on this constraint, the Lagrangian is

$$\mathcal{L} = \int_0^{S_k} e^{-\rho t} \left[w h_t + \sum_{j=1}^{k} \theta_j \right] dt + e^{-\rho S_k} V_{k+1}(h_k) + \lambda[1 - (1-h)e^{-\gamma_k S_k} - h_k].$$

Note that $h_t = 1 - (1-h)e^{-\gamma_k t}$, so $\partial h_t / \partial h = e^{-\gamma_k t}$. The first-order condition for h_k gives that

$$\lambda = e^{-\rho S_k} V'_{k+1}(h_k).$$

Then the envelope theorem tells us that for $h < h_k$,

$$\begin{aligned} V'_k(h) &= \int_0^{S_k} e^{-\rho t} w e^{-\gamma_k t} dt + \lambda e^{-\gamma_k S_k} \\ &= [1 - e^{-(\rho+\gamma_k)} S_k] w(\rho+\gamma_k)^{-1} + e^{-(\rho+\gamma_k)S_k} V'_{k+1}(h_k) \\ &= w(\rho+\gamma_k)^{-1} + e^{-(\rho+\gamma_k)S^*_k(h)} [V'_{k+1}(h^*_k) - w(\rho+\gamma_k)^{-1}]. \end{aligned}$$

Since $S^*_k(h) = -\gamma_k^{-1}[\log(1-h^*_k) - \log(1-h)]$, the time to the next birth is decreasing in h and thus $\exp[-(\rho+\gamma_k)S^*_k(h)]$ is increasing in h. So if we can show that $V'_{k+1}(h^*_k) - w(\rho+\gamma_k)^{-1} \geq 0$ for all $0 \leq k \leq K-1$, we will have that $V''_k(h) \geq 0$. Note also that $V_K = \overline{V}_K$ is linear in h and $V'_K(h) = w(\rho+\gamma_K)^{-1}$.

We will proceed by induction. So assume, for some $k < K$, that $V'_{k+1}(h^*_k) - w(\rho+\gamma_k)^{-1} \geq 0$; we will show that this implies $V'_k(h^*_{k-1}) - w(\rho+\gamma_{k-1})^{-1} \geq 0$. By differentiating $V'_k(h)$ again and rearranging we see that

$$V''_k(h) = (1-h)^{-1}(1+\rho\gamma_k^{-1})[V'_k(h) - w(\rho+\gamma_k)^{-1}]$$

for all $h < h^*_k$. By our inductive assumption, $V'_{k+1}(h^*_k) - w(\rho+\gamma_k)^{-1} \geq 0$, so $V''_k(h) > 0$ and thus $V'_k(h) - w(\rho+\gamma_k)^{-1} > 0$ for all $h < h^*_k$. In particular, for $h = h^*_{k-1}$, we have

$$V_k'(h_{k-1}^*) > w(\rho+\gamma_k)^{-1} \geq w(\rho+\gamma_{k-1})^{-1}.$$

□

A.2 Sufficient Conditions for Nonzero Spacing

If one's preferences for the marginal child are strong enough, it will be optimal to have another birth immediately. Similarly, if those preferences are weak enough, it will never be optimal to have another birth (or, in some cases, to wait so long that *subsequent* interbirth intervals will be zero). To describe the set of parameters such that these corner solutions do not obtain, we first need to describe how the marginal value of human capital, $V_{k+1}'(h)$, depends on preferences $(\theta_k)_{k=1}^K$.

LEMMA 3. *Marginal increases in the payoff from child j affect the utility of a woman of parity k and with human capital h as follows:*

$$\frac{\partial}{\partial\theta_j}V_k(h) = \begin{cases} \rho^{-1} & \text{if } j \leq k \\ \rho^{-1}e^{-\rho S_k^*(h)}e^{-\rho(T_j^*-T_{k+1}^*)} & \text{if } j > k \end{cases}$$

Proof of Lemma 3. First let's establish that for given k, and any $j \leq k$,

$$\frac{\partial V_k}{\partial\theta_j}(h) = \rho^{-1}.$$

Clearly this is true for $k = K$. Then, if the claim holds for $k+1$, it holds for k because, by the envelope theorem,

$$\frac{\partial V_k}{\partial\theta_j}(h) = \rho^{-1}[1-e^{-\rho S_k^*(h)}] + e^{-\rho S_k^*(h)}\frac{\partial V_{k+1}}{\partial\theta_j}(h_k^*) = \rho^{-1}$$

where the second equality uses the inductive hypothesis that $\frac{\partial V_{k+1}}{\partial\theta_j}(h) = \rho^{-1}$ for $j \leq k < k+1$. Next, if $j > k$ we can again use the envelope theorem to get that

$$\begin{aligned} \frac{\partial V_k}{\partial\theta_j}(h) &= e^{-\rho S_k^*(h)}\frac{\partial V_{k+1}}{\partial\theta_j}(h_k^*) \\ &= e^{-\rho S_k^*(h)}e^{-\rho S_{k+1}^*(h_k^*)}\frac{\partial V_{k+2}}{\partial\theta_j}(h_{k+1}^*) \\ &= e^{-\rho S_k^*(h)}e^{-\rho S_{k+1}^*(h_k^*)}\dots e^{-\rho S_{j-1}^*(h_{j-2}^*)}\rho^{-1} \\ &= \rho^{-1}e^{-\rho S_k^*(h)}e^{-\rho(T_j^*-T_{k+1}^*)} \end{aligned}$$

as required. □

Returning to Lemma 3 and differentiating with respect to h, we can see that $V_{k+1}'(h)$ does not depend on θ_j for any $j \leq k+1$. Since the right-hand side of (5) is decreasing in h, a solution h_k^* exists and lies in $(0, h_{k+1}^*)$ if and only if θ_{k+1} lies within certain bounds. Iterating over parities we have the following:

LEMMA 4. *For $k \leq K-2$, let*

$$\underline{\theta}_{k+1}(\theta_{k+2},\dots\theta_K) = (\gamma_k-\gamma_{k+1})(1-h_{k+1}^*)V_{k+1}'(h_{k+1}^*),$$

and let

$$\overline{\theta}_{k+1}(\theta_{k+2}, \ldots \theta_K) = (\gamma_k - \gamma_{k+1})V'_{k+1}(0).$$

If $\underline{\theta}_{k+1} < \theta_{k+1} < \overline{\theta}_{k+1}$ *for all* $0 \leq k \leq K-2$*, and* $(\gamma_{K-1} - \gamma_K)V'_K(0) > \theta_K > 0$*, then*

$$0 < h^*_0 < h^*_1 < \ldots h^*_{K-1} < 1,$$

so birth spacing will be nonzero and finite at all parities.

References

Adda, Jérôme, Christian Dustmann, and Katrien Stevens. 2017. "The Career Costs of Children." *Journal of Political Economy* 125 (2): 293–337. https://doi.org/10.1086/690952.

Barmby, T., and A. Cigno. 1990. "A Sequential Probability Model of Fertility Patterns." *Journal of Population Economics* 3 (1): 31–51. https://doi.org/10.1007/BF00160416.

Beaujouan, Eva. 2020. "Latest-Late Fertility? Decline and Resurgence of Late Parenthood Across the Low-Fertility Countries." *Population and Development Review* 46 (2): 219–47. https://doi.org/10.1111/padr.12334.

Blackburn, McKinley, David E Bloom, and David Neumark. 1993. "Fertility Timing, Wages, and Human Capital." *Journal of Population Economics* 6 (1): 1–30. https://doi.org/10.1007/BF00164336.

Cigno, Alessandro, and John Ermisch. 1989. "A Microeconomic Analysis of the Timing of Births." *European Economic Review* 33 (4): 737–60. https://doi.org/10.1016/0014-2921(89)90023-8.

Cohen, Alma, Rajeev Dehejia, and Dimitri Romanov. 2013. "Financial Incentives and Fertility." *Review of Economics and Statistics* 95 (1): 1–20. https://doi.org/10.1162/003465303772815727.

Happel, S. K., J. K. Hill, and S. A. Low. 1984. "An Economic Analysis of the Timing of Childbirth." *Population Studies* 38 (2): 299. https://doi.org/10.2307/2174078.

Heckman, James J, and James R Walker. 1990. "The Relationship Between Wages and Income and the Timing and Spacing of Births: Evidence from Swedish Longitudinal Data." *Econometrica* 58 (6): 1411–41. http://www.jstor.org/stable/2938322.

Kohler, Hans-Peter, Francesco C. Billari, and Jose Antonio Ortega. 2002. "The Emergence of Lowest-Low Fertility in Europe During the 1990s." *Population and Development Review* 28 (4): 641–80. https://doi.org/10.1111/j.1728-4457.2002.00641.x.

Malkova, Olga. 2018. "Can Maternity Benefits Have Long-Term Effects on Childbearing? Evidence from Soviet Russia." *Review of Economics and Statistics* 100 (4): 691–703. https://doi.org/10.1162/rest_a_00713.

Parent, Daniel, and Ling Wang. 2007. "Tax Incentives and Fertility in Canada: Quantum Vs Tempo Effects." *Canadian Journal of Economics* 40 (2): 371–400. https://doi.org/10.1111/j.1365-2966.2007.00413.x.